Washington State Mathematics Standards

Review and Recommendations

Submitted July 19, 2007

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Executive Summary

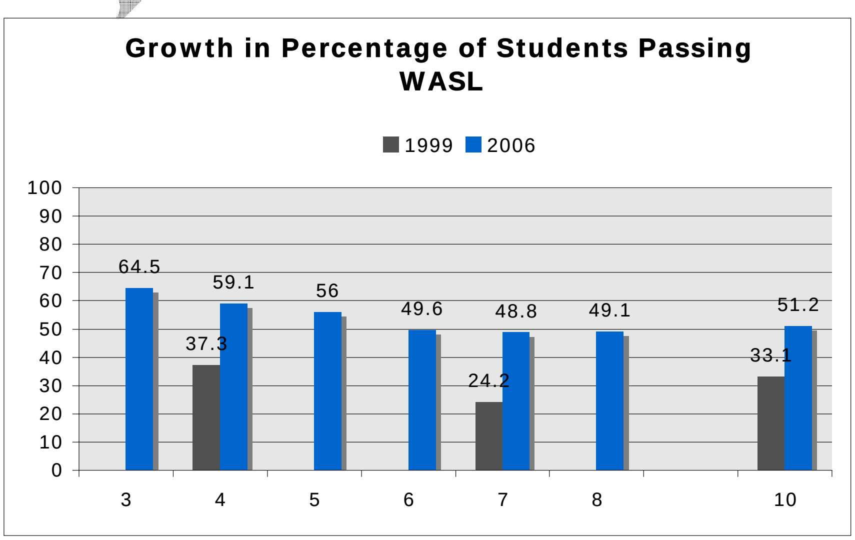
At the request of the State Board of Education, Strategic Teaching has spent the past several months looking at Washington mathematics standards. Using teams of accomplished reviewers, we compared the Washington standards to those in several key states as well as to standards published by key national groups and international expectations.

Using a set of nine rubrics (Appendix A) Strategic Teaching evaluated the content, rigor, specificity, clarity, depth, grade-to-grade coherence, measurability, accessibility, and balance in Washington State Mathematics Standards. We evaluated each of these characteristics using a mix of item-by-item comparisons and a global review of the standards document.

The bottom line is that Washington's math standards need to be strengthened. If mathematics is the gateway to student success in higher education and the workplace, Washington is getting too few of its students to and through the door.

Compared to other higher-achieving states and countries, Washington is not expecting enough of its students. There is insufficient emphasis on key mathematical content. Some key math should be taught earlier in a student's schooling, and some key math is simply missing. Washington does not provide sufficient clarity in its math expectations and does not ensure that Washington students learn the critical algorithms — math rules — that they need to succeed.

And the standards do not provide sufficient clarity of *how well*



students are expected to learn math. For example, the standards often call for student "understanding" rather than a demonstration that a student can actually *use* the math to calculate, estimate, or solve a problem.

This is a harsh assessment. To be sure, there are good qualities in Washington's mathematics standards including well-defined and developed mathematical processes[i] and some well-developed strands, such as Algebra in the elementary years.

Washington is moving in the right direction. The number of students passing the state's tests in math has increased. About 61 percent of the students who took the Washington Assessment of Student Learning (WASL) in June have now passed.

In this report, we present seven recommendations that, if implemented, will provide greater clarity about what is expected of students in each grade, provide more explicit guidance to educators about what to teach when, and ultimately result in more Washington students succeeding in mathematics. Our recommendations are:

1. Set higher expectations for Washington's students by fortifying content and increasing rigor.

2. Prioritize topics to identify those that should be taught for extended periods at each grade level.

3. Place more emphasis on mathematical content and standard algorithms.

4. Write Essential Academic Learning Requirements (EALRs) that clarify grade-level priorities and reflect both the conceptual and procedural sides of mathematics.

5. Increase the clarity, specificity, and measurability of the Grade Level Expectations (GLEs).

6. Create a standards document that is easily used by most people.

7. Include a mathematician, a curriculum specialist, and an effective teacher on the Office of Superintendent of Public Instruction's (OSPI) Standards Revision Team.

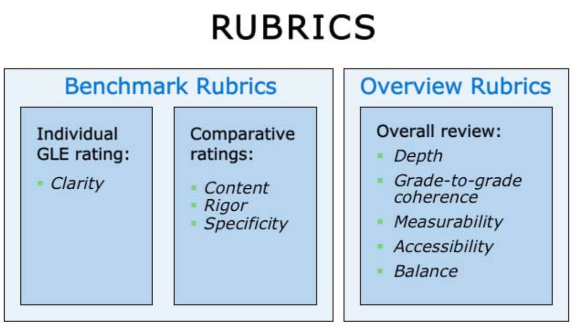
Process

A set of nine four-point rubrics were used to compare and judge Washington's Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs). Appendix B contains summaries of the rubrics. The rubrics, written to examine traits identified by the State School Board, were designed to answer the following questions:

- *Content: Does Washington include the same mathematical content as other, well-respected standards documents?*
- *Rigor: Is the content present at the same grade levels? Are students expected to apply that content in demanding ways?*
- *Specificity: Are the GLEs written with the same amount of detail as other documents?*
- *Clarity: Is it easy to understand what the GLEs mean?*
- *Depth: Are important math topics fully developed?*
- *Grade-to-grade coherence: Do topics develop logically and sequentially over grade levels?*
- *Measurability: Can the GLEs be assessed?*
- *Accessibility: Are Washington's standards easy to use for as many people as possible?*
- *Balance: Is it clear that mathematical content and algorithms, conceptual understanding, and mathematical processes are present in Washington's standards?*

The first three of these traits were evaluated by comparing GLEs to standards in other documents. The documents used for comparison are (1) California State Standards, (2) Massachusetts State Standards, (3) Indiana State Standards, (4) Singapore Curriculum, (5) Finland Standards, (6) *Curriculum Focal Points,* (7) National Assessment for Education Progress, (8) American Diploma Project, and (9) the Washington College Readiness Mathematics Test. Appendix A details the documents used at different grade levels.

Clarity was judged by giving each GLE a score between "1" and "4," with "4" being high, using a set of rubrics created for this project.



The last five characteristics were evaluated from a global perspective after a thorough examination of Washington State Mathematics Standards.

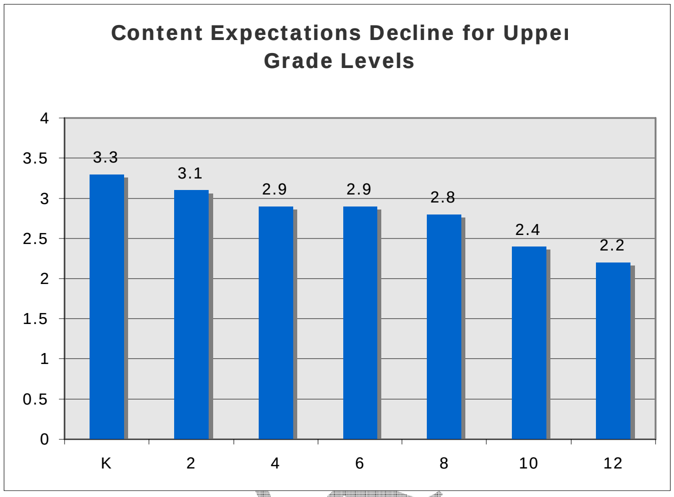
This chart illustrates the three approaches to evaluating Washington standards.

More than 21,000 evaluations of GLEs and comparisons between GLEs and exemplar documents were made during the process. Then the mode, mean, and a weighted mean — designed to reveal the extent to which topics are missing in the GLEs — were identified for each characteristic at each grade level.

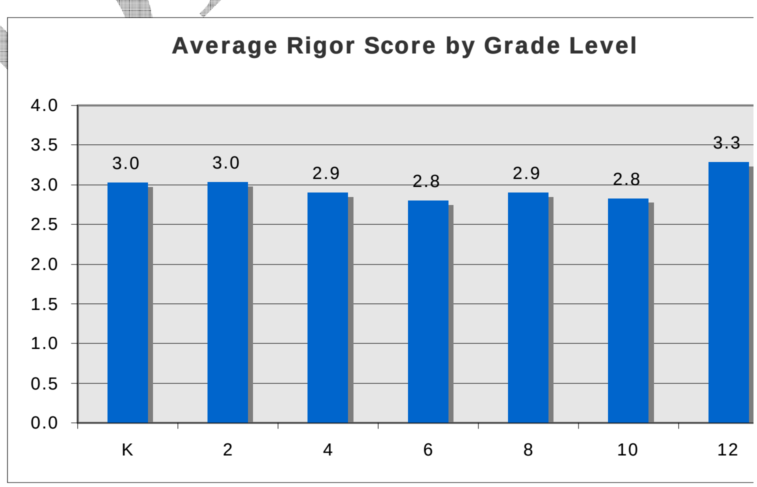
Six reviewers (Appendix C), working in teams of two, completed the reviews and then worked to come to consensus on the score points. The reviewers were chosen because of their knowledge of mathematics, their grade-level expertise, and their experience with standards.

Our Findings

The mode, mean, and weighted mean ("1's" are double-counted in the mean to show how often Washington scored at the lowest level) were determined for the scores from the four-point rubric. The following charts show the average of the scores from all of the documents scored at each grade level.

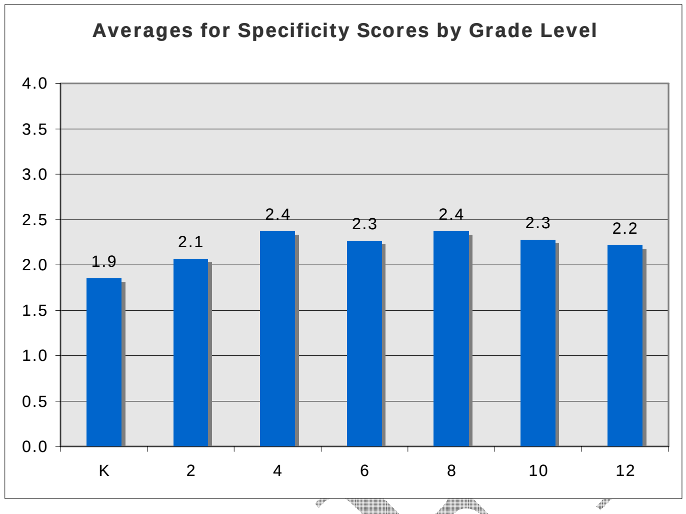


Today, Washington expects less of its students than do other states and high-achieving countries. During the elementary years, Washington students see about 75 percent of the content in the other standards documents that were reviewed. By grade 12 that drops to just over 50 percent.

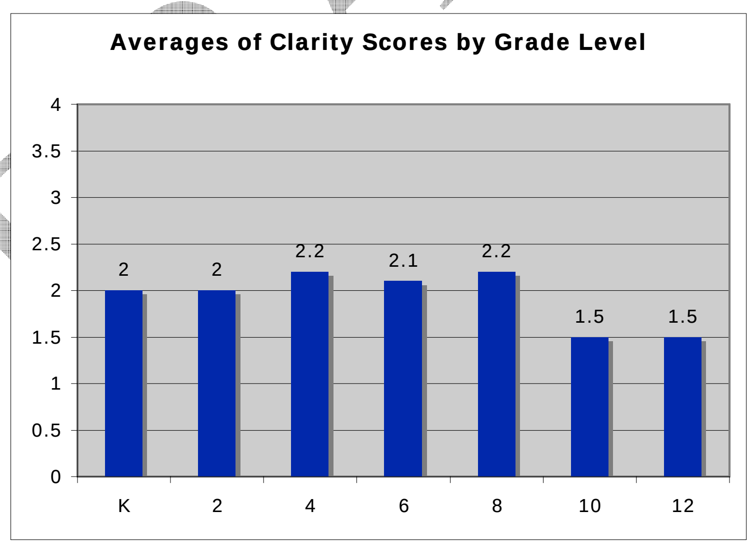


Washington's rigor score hovers around "3" across all grade levels when compared to the other examined documents.

Generally, this means that similar math content is present at the same grade levels, but Washington's students are expected to do less with that content.



Washington's GLEs are a lot less specific than standards in other documents. Sometimes there is less detail about the content, sometimes there is less detail about what students should do with the content, and sometimes both areas are vague.



Clarity scores are low, ranging from "1.5" to "2.2." Both "1" and "2" reflect GLEs that are not clear. The score of "2" is given when the GLE is unclear, but the examples under the GLE help. A score of "1" is given when, even after reading the examples, we do not know what the GLE means.

The lack of clarity stems partly from GLEs that are written at the general, rather than specific, level. Another cause is Washington's repeated use of the verb "understand," which is open to multiple interpretations.

The following examples illustrate strong matches, "4s", between a GLE and an exemplar standard for specified traits. These are the kinds of matches we want to see more often. Please note that a "4" score on one trait does not mean that the match would score "4" on other traits. In the first example content rates "4", but rigor scores "3" (the content is at the same grade level, but Washington expects students to do less with the content); specificity scores "2" (Massachusetts is much more detailed); and clarity scores "2" (it is vague, but the examples clarify the meaning).

Content: The GLE on the left matches the Massachusetts standard on the right.

WA grade 8, GLE 1.4.1	**Massachusetts grade 8, 8.D.4**
Understand the concept of compound events.	Use tree diagrams, tables, organized lists, basic combinatorics ("fundamental counting principle"), and area models to compute probabilities for simple compound events, e.g., multiple coin tosses or rolls of dice.

Washington earns a "4" for matching the content in the Massachusetts standard because both standards are about the same math topic: compound events.

Rigor: The GLE on the left matches the California standard on the right.

WA grade 4, GLE 1.5.4	**California grade 4, A.1.1**
Use a single variable to write expressions and equations that represent situations involving multiplication and division of whole numbers.	Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).

Washington earns a "4" for matching the rigor in the California standard because the content is at the same grade level in both documents and Washington has similar expectations about what students should do with the content.

Specificity: The GLE on the left matches the Massachusetts standard on the right.

WA grade 8, GLE 1.4.4	**Massachusetts grade 6, PS 1.2**
Identify clusters and outliers in data and determine effects on the measures of central tendency.	Understand how additional data added to data sets may affect these computations of measures of central tendency.

Washington earns a "4" for matching the specificity in the Massachusetts standard because the two standards are written at about the same level of detail.

The traits of depth, grade-to-grade coherence, measurability, accessibility, and balance are given overall scores. They are judged by thoroughly examining the Washington State Mathematics Standards document as a whole.

Washington earns "1's" across all of these categories. Missing (computation) and scattered (fractions) core content forces a low score for depth or development of math content. Without the core content, little else matters.

Areas of Concern

- *GLEs are not articulated by high school math course. From an outsider's perspective, Washington's standards seem useless to secondary teachers.*
- *Understanding mathematics outweighs concrete mathematics. People disagree about what is important in teaching math. One can think about this as a continuum of viewpoints that range from the purely "conceptual" on one end to the purely "procedural" on the other. Washington's standards are too far on the conceptual side.*
- *EALRs are poorly organized. Each EALR has multiple components; each component has multiple GLEs, which are further organized by grade level; and most GLEs have multiple examples. The five layers are redundant and confusing.*
- *Algorithms are missing or sidestepped in Washington's standards. From a teaching support perspective, the area most in need of strengthening is the lack of a thorough and connected set of computation algorithms across all strands and all grades.*

- *Verbs, which define what students are to do with the math content, are often too broad and not measurable. If a teacher is expected to teach in a standards-based environment, then the teacher has to be able to make judgments about the extent to which students have met the standard. Verbs that define observable, measurable activities make this possible.*
- *Washington's standards rely on examples. Many, perhaps most, of the GLEs require the user to read the examples to figure out what the standard means and requires.*
- *GLEs for grades 11 and 12 appear to have been lifted predominately from the Washington College Readiness Standards with a dash of NEAP for seasoning. They feel more like a convenient solution to a requirement than a thoughtful effort to identify a threshold math capability.*

Areas of strength

There are positive aspects of Washington's current standards.

- *Conceptual understanding and mathematical processes are well present in the standards.*
- *The grade-to-grade connections are stronger at the elementary grades.*
- *The side-by-side presentation format of the GLEs shows well the interconnectedness and maturation of the math in Washington's standards. This format supports the vertical articulation that is essential to good mathematical instruction and should continue to be offered in addition to other formats discussed later.*
- *The Algebra strand K–8 is strong. It does a good job of early grades articulation.*

Recommendation 1: *Set higher expectations for Washington's students by fortifying content and increasing rigor.*

Washington's standards were first published in 1997. They reflected the then widely embraced standards of the National Council of Mathematics Teachers. Washington was in the forefront as it built an aligned and comprehensive standards, assessment, and professional development system to support student achievement.

Much more is now known about standards that support student learning than a decade ago. Washington is wise to revisit its standards and learn from the work of others.

Well-defined content is the core of a standards document. The content in Washington's standards needs to be strengthened in three ways.

First, the GLEs need to be expanded to incorporate missing content. For instance, the concept of "odd and even" is missing in Washington and present in both Indiana and Massachusetts. This important early concept helps prepare students for division, prime numbers, factorization, and prime roots. Although it is likely that this concept is widely taught by Washington teachers, it should be clearly stated in the GLEs.

Introduce students to core content at earlier grade levels, expand the content to include more advanced topics, and better develop core concepts, especially at the secondary level.

Second, much of the core content in the standards needs to be moved to earlier grade levels. Fractions, for instance, are introduced in Washington at grade 4. Singapore and California fully develop this topic at grade 2.

Another example is that Washington expects students to add and subtract numbers to 18 in grade 2. Singapore introduces adding and subtracting in grade 1 and expects students in grade 2 to add and subtract numbers to 1,000. We are not suggesting that standards from other places should just be adopted wholesale from another document. Singapore, for example, requires students to use the abacus.

Third, some of the topics included in Washington's standards need to be better developed. The quadratic equation, a fundamental tool in solving algebraic equations, exemplifies an underdeveloped topic. Washington references it in its grade 11–12 document but fails to treat it with necessary depth or breadth. Compare Washington's and California's treatments of the quadratic equation.

Washington

- *Apply procedures to solve linear inequalities, quadratic, absolute value, radical, and exponential equations.*

- *Define symbolic notation of functions (linear, quadratic, cubic, simple exponential, and simple rational functions).*

California

- *Students solve a quadratic equation by factoring or completing the square.*
- *Students know the quadratic formula and are familiar with its proof by completing the square.*
- *Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.*
- *Students graph quadratic functions and know that their roots are the x-intercepts.*
- *Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points.*
- *Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.*

The examples, “Exhibitions of Learning”, listed under the GLEs contain some of the important math content included in California. The Standards Revision Team should consider the examples when they revise the standards.

The rigor in Washington’s standards is compromised by two factors.

First, as just discussed, mastery of content is expected late in a student’s career.

Second, the rigor is inadvertently undermined by the overuse of the verb “understand.” “Understand” does not necessarily require students to use math content in sophisticated ways. Yes, students must understand. They also need to do something with their understanding, including solving multistep word problems.

To illustrate, in grade 4, EALR 1, “understand” is used 10 times, “use” three times, “apply” twice, and “read” and “recognize” once each. Contrast this with the same grade level in Indiana. Indiana expects students to read, write, identify, round, order, compare, name, rename, and rewrite.

Recommendations to fortify content and increase rigor:

- ***Include or better develop the content listed below.***
- ***Move appropriate core content to lower grades.***
- ***Replace the verb "understand" with verbs that require students to do demanding work.***

Specific recommendations for kindergarten to grade 8 content:

- *Anchor basic arithmetic operations in the place value system — the foundation of our numbering system.*
- *Strengthen the development of addition and subtraction concepts by linking them to place value, grouping, and regrouping.*
- *Include "fraction readiness" in the early elementary grades.*
- *Place a higher priority on fractions and present them in a mathematically structured, coherent fashion.*
- *Fortify the grade 3 curriculum by developing students' understanding of the concepts of numbers written out using place value by applying the commutative and the distributive property to see how the standard algorithm for multiplication comes about naturally.*
- *Develop, beginning in grade 4, the standard long division algorithm. Long division builds estimation skills and sets students up for a basic concept in calculus: successive approximation. Long division does not always give you a final answer, but each step gives you a better approximation of the answer.*
- *Bring absolute value, the distance from a number to zero on the number line, into the GLEs. It is used to add and subtract negative numbers, find the maximum and minimum of simple problems, and find the distance between points on a coordinate plane, and it is used in algebraic calculations with radicals.*
- *Stress multistep word problems for all K–8 mathematics.*

Specific recommendations for content for all graduating students:

- *Refashion the high school algebra standards to include the concepts contained in the GLE examples for Algebra I. The examples under the GLEs contain concepts and skills worthy* *of GLE status. These should not be suggestions, but requirements.*

- *Develop high school geometry in a coherent fashion with a good introduction to proofs including foundational concepts such as axioms, postulates, and beginning graph theory.*

Specific recommendations for content to prepare a student for college, ready to be successful in Calculus:

- *Include a study of trigonometry supported by basic trigonometric functions used in computations, such as sine, cosine, and tangent.*
- *Reinforce the essential calculus readiness standard of being able to manipulate rational functions by spelling out student requirements for the application of procedures to simplify and evaluate polynomial, rational, absolute value, and radical expressions.*
- *Expand the standards for quadratic equations to include a deep analysis of line of symmetry, max/min for quadratic function, the quadratic formula, and completing the square to determine the attributes of a circle, ellipse, parabola, and hyperbola.*
- *Add complex numbers, polar coordinates, and induction.*
- *Incorporate function composition and inverse functions, matrices, conic sections, advanced statistics, logarithms, and series. These topics are contained in at least two of the following documents important to Washington and its students: Washington College Readiness Assessment, American Diploma Project Benchmarks, and the National Assessment for Education Progress.*

Recommendation 2: *Prioritize topics to identify those that should be taught for extended periods at each grade level and to better show a topic's development over grade levels.*

The existing document, organized by components, shows well how existing topics in GLEs develop over and connect within grade levels. Although there are exceptions — for example, probability in the lower grades — where particular topics are clearly omitted, the vast majority of components exist for every grade level. Now the EALR structure mandates "form over structure" and encourages creating a GLE for every "square," meaning every topic is equally emphasized at every grade level.

The content for a single topic is spread over multiple grade levels and every grade or course includes numerous topics.

When the standards originally were adopted 10 years ago, building concepts over grades and revisiting topics repeatedly were believed to be the best ways to ensure student understanding, remembering, and expertise. This is no longer considered best practice for teaching and learning.[ii]

The revised standards should identify topics that will be taught over extended periods of time during a single school year. These core topics or grade-level “themes” should be clearly defined. Teachers should not have to guess which topics are most critical.

Some non-priority topics should be consolidated and moved to single (or successive) grade levels. Rather than spreading statistics, for example, across all twelve grades, it should be taught for an extended period of time during two or three selected grade levels.

This approach allows topics to have depth and to be fully developed.

Currently there is no way, beyond a teacher’s good sense, to know whether he or she should pay more attention to “mixed numbers, proper and improper fractions, and decimals” or “angle measurement” or “mean, median, and mode” in grade 5.

The structure now also means that single topics get distributed to various nooks and crannies of the document. For example, “fractions” appears to be well developed, but it is so scattered throughout the document that it is difficult to be sure. The structure of mathematics is part of the content of mathematics. To present content properly, the structure of that content needs to be clear.

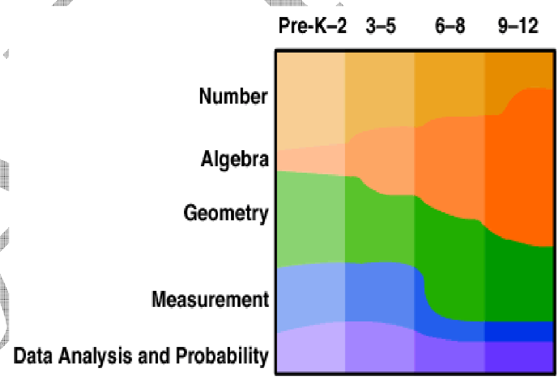
Grade-to-grade coherence—the extent to which a topic’s complexity grows sequentially over grade levels—relates to prioritized topics. Consolidating topics within fewer grades makes the subsequent development more apparent.

We are not suggesting that all topics except the priority topics be excluded. Telling time, for example, is and should be part of a math curriculum. It just should not take up much time in that curriculum. It can be reinforced by bringing it into the priority topics by asking, “How many minutes pass between 9:18 a.m. and 2:15 p.m.?”

At the risk of being repetitive, the heart of mathematics is number sense and computation, and these topics should be emphasized in elementary grades. Place value makes possible arithmetic as we know it. (Think of trying to compute with Roman numerals.) Adding, subtracting, multiplying, and dividing whole numbers, fractions, and decimals without the aid of a calculator and with fluency should be the core of K–5 math. These should be followed in subsequent grades with ratios, rates, proportions, and percentages. Washington should insist on the ability of all students to use all of these skills to solve multistep word and story problems.

There are several resources that offer ways to expand, reduce, and consolidate math topics.

Principals and Standards, published in 2000 by the National Council of Teachers of Mathematics (NCTM), includes this chart to clarify its position that not all topics are equally important at all grade levels.



Curriculum Focal Points (CFP), released by NCTM in September 2006, identifies major topics at each grade level K–8. This document has been well received by people with differing perspectives. *CFP* limited itself to three core topics per grade level and identified how these connected to other topics in math.

Singapore varies the strands that are taught at different grade levels. Some topics are consistent across grade levels, whereas other topics fade away to be replaced with new topics. The chart below shows selected grade levels from Singapore.

Grade 2	Grade 6	Grades 9 and 10
Whole numbers Money, Measurement & Mensuration (the measurement of geometric figures) Statistics Geometry Fractions	Money, Measurement & Mensuration Statistics Geometry	Numbers and Algebra Geometry and Measurement Statistics Probability

California uses Number Sense (Number Theory, Fractions, Computation, etc.); Measurement and Geometry; Algebra and Functions; and Statistics, Data Analysis, and Probability as the organizational structure for grades K–7 and then uses custom strands for the high school math courses. Other states use similar organizational systems.

Specific recommendations to increase the depth and development of mathematical topics:

- *Major themes that require extended teaching time should be identified for each grade level. The topics in need of front-and-center attention for both elementary and secondary are identified in the previous section.*
- *Grade-to-grade coherence of mathematical topics should be apparent with ever more sophisticated content appearing over grade levels.*
- *Additional topics that are more minor in nature should be explicitly included while making it clear they require less classroom time.*
- *Some topics should be consolidated and moved to single grade levels where they will take their turn as areas of emphasis.*

Recommendation 3: *Place more emphasis on mathematical content and standard algorithms.*

Mathematics can be thought of as a three-legged stool: mathematical content including standard algorithms, conceptual understanding, and mathematical processes (communicating, reasoning, problem solving, and connecting math content). Missing any leg cripples the user. Applying content in novel situations without conceptual understanding is close to impossible. Applying conceptual understanding in high-level mathematics without algorithms is impossible.

One of the strengths of the current GLEs lies in the maturity of the process strands. They are thoughtfully constructed and well developed. They also make clear the importance of conceptual understanding with numerous standards such as "Understand the concept of area" from grade 4.

Mathematical content and fluency with standard algorithms, conceptual understanding, and mathematical processes are all necessary. This should be crystal clear in Washington's standards.

Mathematical procedures should not take attention away from mathematical content. Math content is core to a standards document and should be well defined. Mathematical procedures, especially standard algorithms, need to be spelled out.

Many GLEs are similar to this one from grade 4: "Apply strategies and use tools appropriate to tasks involving multiplication and division of whole numbers."

"Strategies" gives the impression that addition and subtraction problems are ad hoc problems. They are not. There are standard procedures, algorithms, that work every time. Let students look for many ways to solve simple arithmetic problems; this builds understanding. Just make sure they know one way that always works.

Phrases such as "use tools appropriate" begs the questions of what kind of tools and under which conditions they should be used. The use and misuse of technology mandates plain direction.

There is a belief that calculators are used in elementary classrooms in lieu of memorizing math facts. There are plenty of anecdotes, but no research to support this. It is known that American teachers use calculators in elementary schools more often than their counterparts in high achieving countries[iii] but that does not necessarily equate to teaching students to use calculators rather than memorizing math facts and standard algorithms.

Calculators have value in the elementary classroom, but not to replace computation. Students need to add, subtract, multiply, and divide without a calculator.

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At the secondary level, again, the use of technology should not circumvent student fluency with algorithms. Graphing calculators and spreadsheets are powerful tools that help students make connections among different representations. Many concepts — range, roots, and optimum values — come alive with technology. A variety of software now available helps students understand systems of linear equations, quadratic functions, absolute values of quadratic functions, conics, and more. Still, the use of technology should not replace a student's ability to solve problems manually using standard algorithms.

Nowhere are students asked to memorize their addition and multiplication facts or to use standard algorithms with fluency and efficiency. Algorithms should be taught and listed as standards.

Recommendations to clarify the importance of all aspects of mathematics:

- ***Require students to memorize math facts and use standard algorithms fluently.***
- ***Retain conceptual understanding, but do not let it overshadow content and algorithms.***
- ***Retain the richness of the process strands, in part, by thoughtfully embedding it into important content.***

Recommendation 4: *Write EALRs that clarify grade-level priorities and reflect both the conceptual and procedural sides of mathematics.*

This recommendation flows from the previous two. Washington's current structure, with five EALRs that are consistent across all grades, needs to be modified.

The new structure should be built around EALRs that reflect the decisions made by the Standards Revision Team about the topics that will receive extended focus at each grade level. These topics should reflect the core mathematics identified in Recommendation 1. The goal is to identify a limited set of topics that can be taught and learned well within a school year.

The topics or strands defined by the EALRs need not be consistent across all grade levels. It is more important that they assign critical content to specific grade levels and that they ensure that related content is together. A document that leaves no room for confusion about what is important takes priority over a document with symmetry.

Create EALRs that reflect top-priority content and that balance conceptual understanding and procedural faculty with that content.

The new EALRs will structure a standards document that clarifies the importance of math content and standard algorithms for that content.

This requires unpacking EALR 1 to create separate topical strands that vary across grade levels. And it requires collapsing EALRs 2–5 into a single strand called something like "Mathematical Reasoning."

Additionally, conceptual understanding and math processes should be fortified by embedding them into the content strands. Processes need to explicitly connect to appropriate content.

The new structure will better translate into a state assessment that aligns with grade-level priorities and more fairly assesses students on what they know. The standards serve as the blueprint for the Washington Assessment of Student Learning

(WASL). The current structure distorts the assessment of grade-appropriate topics. The current even distribution of content means a student's WASL score comprises as much Algebra in grade 2 as in grade 10 and as much number sense in grade 8 as in grade 1. (Although the number of items varies, the percentage each strand counts in a student's score is consistent across grade levels.)

Specific recommendations to create a new structure defined by revised EALRs:

- *Unpack EALR 1 by identifying the priority topics for grade levels and using those to create an expanded set of EALRs about mathematical content.*
- *Use the new EALRs to create strands that vary across grade levels and that reflect priority content.*
- *Embed standard algorithms into the content strands.*
- *Collapse EALRs 2–5 into a single rich process strand.*
- *Embed conceptual understanding, process strands, the memorization of math facts, and fluency with standard algorithms into the content strands.*

Recommendation 5: *Increase the clarity, specificity, and measurability of the GLEs.*

Many of the current GLEs are open to numerous interpretations. This may have been an intentional attempt to allow for rich interpretation by teachers. However, it compromises the intent of standards, which is to define clearly what should be taught, when.

For example, a Grade 1 GLE includes the following language: "Apply strategies and use tools appropriate to tasks involving addition and subtraction of non-negative decimals or like-denominator fractions" Reasonable people could well disagree on what this means.

We have already discussed "strategies," and "tools." "Strategies" are not algorithms and algorithms are needed. And, the use and misused of technology requires plain direction.

In too many GLEs, like the previous example, the verbs are broad and unobservable and the content is ill defined. Simply put, there is not enough detail — specificity — for the GLEs to be clear. Certainly this is not always the case, but it is persistent and needs to be fixed.

The following chart shows how Washington's standards compare to other documents regarding verbs and content. Verbs are underlined to make the differences apparent.

WA	CA	MA	IN	Curricular Focal Points
1.1.1 Understand the concept of number to at least 31.	**N 1.2** Count, recognize, represent, name, and order a number of objects (up to 30).	**K.N.1** Count by ones to at least 20. **K.N.2** Match quantities up to at least 10 with numerals and words. **K.N.8** Estimate the number of objects in a group and verify results.	**K.1.3** Know that larger numbers describe sets with more objects in them than sets described by smaller numbers. **K.1.6** Count, recognize, represent, name, and order a number of objects (up to 10). **K.1.8** Use correctly the words one/many, none/some/all, more/less, and most/least.	**N 1** Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. **N 2** They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.

It is not easy to create a set of standards with enough — but not too much — detail. Striking the right balance between broad statements that are open to multiple interpretations and overly detailed standards that reduce mathematical topics into a set of tiny, discrete skills is difficult.

Adding sample problems to the GLEs is one way to clarify meaning. A sample problem is a specific problem that illustrates the intention of the standard. These are different than the existing examples, which are really descriptions of classes of problems. Presently the examples contain important content that belongs at the GLE level.

Indiana skillfully uses sample problems as is shown by its grade 4 standard, “Understand the special properties of 0 and 1 in multiplication and division. *Example: Know that* $73 \times 0 = 0$ *and that* $42 \div 1 = 42$.”

The old adage that “things should be as simple as possible, but no simpler” applies here.

The good news is that addressing specificity and clarity by adding detail to the content and replacing “understand” with observable, measurable, action verbs creates a document that is measurable. This creates a set of standards that teachers can use to gauge student progress and testing companies can use to design appropriate test items.

Specific recommendations for clarity, measurability, and specificity are:

- *Define the parameters of the content with details that provide clear guidance.*
- *Use specific verbs that describe what students should be able to do with the content.*
- *Reduce the use of the verb “understand” and replacing it with action verbs that describe behaviors.*
- *Define when and under what circumstances technology, especially calculators, should be used. That calculators should not be used in lieu of computational mastery must be plain.*
- *Embed sample problems within the GLE as needed for clarity.*

Recommendation 6: *Create a standards document that is easily used by most people.*

The existing standards are written with little mathematical jargon or unnecessary mathematical vocabulary. Still, their organization and presentation is unwieldy.

The GLEs should be available in multiple formats.

The current format is helpful to curriculum specialists who work across grade levels, groups of teachers when they meet to talk about how content builds across grade levels and the implications of that on teaching, and individual teachers when they want to see what is taught before and after their grade level; because of this, it should continue to be an option.

It is less helpful to other audiences and for other purposes.

Not enough has yet been said about how the current structure impacts the secondary level. Currently, at least to the outsider, it is impossible to tell what standards should be taught in traditional courses, such as Algebra and Geometry. How does a teacher isolate the standards for which they are responsible? Do teachers diligently identify and teach those GLEs that seem to fit into the courses they teach? Do hundreds of schools pull together teams of teachers that allocate each GLE to a specific course?

The grade-level format seems to be a better fit for Integrated Math I, II, and III, but it is not certain there is a perfect correlation. How would one know?

The Standards Revision Team must address this situation and make clear what should be happening in specific secondary courses. Massachusetts offers a helpful approach. Their secondary standards are offered both by grade level and by course. The numbering system they use matches standards in the grade-level format to the standards in specific courses. It is clear that a student taking Integrated Math series of I, II, and III will learn the same materials as a student taking Algebra, Geometry, and Algebra II.

Primarily, offer the GLEs in the following ways:

- *By single grade levels K–12 to serve elementary teachers, secondary teachers of Integrated Math, and parents interested in what their children are learning.*
- *By secondary courses so that it is clear which standards should be taught and learned in specific classes.*
- *As is, showing multiple grades, to support vertical articulation and mathematics instruction.*

Also consider offering the versions, which can be developed and offered over time:

- *By strands so that the development of Algebra, for example, can be traced over multiple grade levels.*
- *In different levels of complexity so that the public can access broadly, simply written standards to answer such questions as, "What will my second-grade student learn this year?" A second, more detailed level could be geared toward*

educators. A third, even more specific level could be created for the testing contractor and could include such things as assessment limits on types and sizes of numbers.

- *With different illustrations and example problems for various audiences.*

All versions and all levels should be available to everyone. This is about creating options and making those options available through a well-designed menu of choices to anyone who is interested.

Renumber the GLEs so they are more intuitive. The Massachusetts system tells the user at a glance how the standard they are looking at fits into the schema. For instance, 4.N.3 is a grade 4 standard from the Number Sense strand and it is the third standard listed.

Identifying, within the GLEs, words included in the glossary will make the standards easier to use. Providing definitions within the GLE text occasionally makes sense, but generally, noting terms that can be found in the glossary — perhaps with an asterisk — serves the purpose without adding unnecessary bulk.

Minor changes — more technical than substantive — would make the document more transparent and useable.

Mark mathematical topics needed to be “college ready” so it is easy to differentiate between rigorous content for *all* students and content needed by those who want to go beyond minimum expectations. This suggestion just clarifies within the Washington State’s Mathematics Standards document the implementation of Recommendation 1.

Including and marking advanced mathematical topics should not encourage tracking: this is not about limiting the opportunity of any interested student to take high-level math courses. Able students should be encouraged to take as much math as possible. Math is useful and opens doors. Encouraging every student to take as much math as possible creates opportunities.

Additionally, relatively minor changes — more technical than substantive — would make the document more transparent and more useable.

Use current online technology to offer a rich, multilayered document. Include hyperlinks to definitions, example problems, released test items, explanations of content, and more.

1. Essential Academic Learning Requirement
2. Component
3. Grade Level Expectations
4. Topic
5. Examples

Reduce the number of "levels" to reduce confusion. As previously discussed, Washington's standards are now organized into five EALRs. Each EALR has multiple components, each component has multiple GLEs (which are further organized by grade level), and most GLEs have multiple examples.

One way to think of this hierarchy is that each EALR stands for what most states call content "strands." The components are like "sub-strands," the GLEs are analogous to what most states call "standards," and the examples serve to amplify the standard. Another way to think about the EALR hierarchy is to visualize each EALR as an overarching idea with the components as strands and the GLEs as standards, etc. Either way it is confusing to users.

Most standards documents have three levels, although the American Diploma Project has with two. Some sort of simple hierarchy with an intuitive structure is the backbone of a document that is easily used by as many people as possible.

Consider aligning — precisely, standard by standard — with high school assessments such as SAT/PSAT/ACT/AP/End of Course assessments and college placement tests and, if possible, adopted text materials. This information does not have to clutter the main document, but would be one of the supplemental versions available.

Recommendations to increase accessibility:

- ***Offer the standards in a variety of formats. At the minimum make them available by grade level, by subject course name, and by grade bands.***
- ***Renumber the GLEs in a way that is more intuitive.***

- ***Embed indicators that alert the reader when a word is included in the glossary.***
- ***Use online technology with embedded hyperlinks that allow users to access additional information as they wish.***
- ***Include and mark "college-ready" standards that go beyond expectations for all students.***
- ***Reduce the number of "levels" in the current document.***
- ***Use online technology to offer hyperlinks to additional samples, explanations of content, and other helpful content.***
- ***Consider offering other formats such as content strand or different levels of complexity, which can be developed over time.***
- ***Consider aligning standards to pertinent assessments and curricular materials, as a supplemental or online option.***

Recommendation 7: *Include a mathematician, a curriculum specialist, and an effective teacher on OSPI's Standards Revision Team.*

The Office of Superintendent of Public Instruction (OSPI) has the responsibility of revising Washington's mathematics standards within a few months. This is a short timeframe for such work. The best way to go about this is to create small three-person teams for each grade band.

The small writing teams need to include the people most necessary to the success of the work: a mathematician, a teacher from the relevant grade band, and a curriculum specialist. One person with extensive standards experience in multiple states should facilitate, organize, and coordinate the work to be sure there is consistency across grade bands.

Other perspectives — from business community members, College Transition Project members, parents, mathematics educators, college educators, industry leaders, child development experts, mathematics researchers — are valuable and should also be heard. But if OSPI is to keep the ambitious schedule set by the legislature, the writing teams must be small.

In lieu of inclusive writing teams, OSPI should convene formal focus groups to listen to other stakeholders. Targeted focus groups — one for mathematicians, mathematics educators, and mathematics researchers; one for the business community that includes representatives from math-intensive fields; one for teachers; and one for parents — should inform the standards revision process.

Additionally, the draft standards should be sent to groups, such as the College Readiness Project, that have special interests in the standards and valuable input to offer.

All of this would be in addition to normal feedback loops, such as posting the document on the Internet.

Specific recommendations to rewrite the standards:

- *Create three-person teams for each grade band and have the three writing teams overseen by a knowledgeable coordinator.*
- *Hold at least four focus groups for critical stakeholders.*
- *Send the draft document to interested groups.*
- *Offer normal feedback loops such as online posting.*

Conclusion

We offer a clear and specific set of recommendations for revising Washington's mathematics standards. We believe that these recommendations will produce a strong set of standards to serve as the base for the work outlined in the Joint Mathematics Action Plan. As we said in the beginning, Washington is moving in the right direction. The revised standards are the first step in accelerating that progress.

Appendix A

DETAILS OF DOCUMENTS USED FOR COMPARING GLEs BY GRADE LEVEL

	K	2	4	6	8	10	12
CA	**K**	**Grade 2**	**Grade 4**	**Grade 6**	**Algebra I**	**Algebra I and Geometry**	**Algebra II**
MA	Grade band pre-K and K	Grade band 1–2	Grade band 3–4	Grade band 5–6	Grade band 7–8	Grade band 9–10	Grade band 11–12
IN	K	2	4	6	8	Integrated 1 and 2	Integrated 3
SG	Doesn't exist at this grade	Primary 2	Primary 4	Primary 6 EM1/EM2 iv	"O" levelv Secondary 1	"O" level Secondary 2	"O" level Secondary 3/4
FI	Doesn't exist at this grade	Grade 2, "Core content" and "Measures of good perform-ance"	Content is organize d in grade band 2–5	Content is organized in grade band 6–9	Compared to Grade 8 assess — when working from GLE examine grade band 6–9 and do not score for rigor	Not available at time of review	Not available at time of review
CFP	Break the paragraphs into sentences and skip "connec-tions"	Break the paragraphs into sentences and skip "connec-tions"	Break the paragraphs into sentences and skip "connec-tions"	Break the paragraphs into sentences and skip "connec-tions"	Break the paragraphs into sentences and skip "connec-tions"	Doesn't exist at this grade	Doesn't exist at this grade

NAEP	Doesn't exist at this grade	Doesn't exist at this grade	Grade 4	Doesn't exist at this grade	Grade 8	Doesn't exist at this grade	Grade 12
ADP	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Benchmark here
WA test	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Doesn't exist at this grade	Benchmark here

Appendix B

RUBRIC SUMMARIES

Key ideas, by score point, for Content, Rigor, Specificity, and Clarity

Score	Content	Rigor	Specificity	Clarity
4	Regardless of grade level *or* cognitive demand; Content matches content. Note: Wording may not match exactly.	The content and grade level match *and* the GLE Bloom's Taxonomy level is at least as demanding: 1. Remembering 2. Understanding 3. Applying 4. Analyzing 5. Evaluating 6. Creating.	The level of detail in the Benchmark matches the detail in the GLE. Matching grain size.	Reader finds total clarity. Content and parameters of content well defined. Standard well written and clear. No ambiguity.
3	Regardless of grade level or cognitive demand; Content is connected but not an exact match.	The content matches and the grade levels match … *but* the GLE taxonomy level is lower than the Benchmark.	Missing detail matches logical assumptions Similar, but not matching, grain size.	Understandable; stands alone; good vocabulary. May have small disputes in meaning.
2	Regardless of grade level or cognitive demand; Content is present, but is not as sophisticated.	The GLE content is at a lower grade … … EVEN WHEN the GLE taxonomy level is equal to or higher than the Benchmark.	The level of detail of the GLE differs in regard to content or performance demands of the Benchmark. Different grain size.	Multiple possible meanings; vague wording. Examples clarify somewhat.
1	Content *not* present in GLE.	The GLE content is at a lower grade … and the taxonomy level is lower than the Benchmark.	The topic or sub-topic detail in the GLE does not approach the scope or topic detail in the Benchmark. Dissimilar scope.	Unsure of meaning even after Examples and discussion. Incorrect use of mathematical vocabulary automatically scores "1."
NA	Missing Content.			Missing Content.

Key ideas, by score point, for Depth, Grade-to-Grade Coherence, Measurability, Accessibility, and Balance

Score	Depth	Grade-to-Grade Coherence	Measurability	Accessibility	Balance
4	All major topics are included and are fully developed.	Content systematically increases in complexity. Prerequisites are in place. Consistent cognitive demand across strands with evenly distributed content across grade levels.	Observable content, skills, behaviors. Clear expectations for mastery.	Useable by as many people as possible. Format is self-explanatory. Content available in a variety of ways.	Addresses all three aspects: math concepts, algorithms, and why the algorithms work.
3	All major topics are included *but* a few topics are underdeveloped.	Content increases in complexity but there are missing prerequisites; *or* Inconsistent expectations around cognitive demand; *or* Problems with content distribution.	Clearly defined content. Murky expectations of performance.	Generally understandable; Only one format.	Addresses both conceptual understanding and execution.
2	Most major topics are included *and* a few topics are underdeveloped.	Minor breakdowns in content sequence. Two or more of the problems in "3."	Content defined. Verbs are difficult to assess.	Understandable to teachers, but difficult for others.	Both concepts and algorithms are present, but one dominates.
1	Numerous topics are missing.	Serious inconsistencies or interruptions in the content sequence.	Content vaguely defined. Unobservable verbs.	Difficult to understand by everyone.	One aspect is excluded.

Appendix C

MEMBERS OF THE STANDARDS REVIEW TEAM

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Connie Colton is a master teacher in the Omaha Public School District. She has been teaching mathematics at the secondary level for the past 15 years. She holds a bachelor's degree in secondary education with a field endorsement in mathematics from the University of Nebraska at Omaha. She has a master's degree in secondary education with an emphasis in mathematics and curriculum development and is currently a National Board Certification candidate.

Rhonda Naylor, M.A., a National Board Certified Teacher in early adolescence mathematics, taught sixth- through eighth-grade students for 30 years. She has a bachelor's degree in elementary education, a master's in mathematics education, and certification in secondary education from the University of Colorado at Boulder.

Sandy Sanford, Ed.D., has been teaching in California public schools since 1989. He has taught at all levels and ended his public school career as district administrator of assessment, research, and evaluation. In 2000, Dr. Sanford created a laboratory to develop better methods for gathering, analyzing, reporting, and using assessment data to guide the instructional process in a standards-based educational environment. Sandy has an M.S. in systems management from the University of Southern California and an M.Ed. and an Ed.D. from Azusa Pacific University.

Eric J. Rawdon, Ph.D., is an assistant professor at the University of St. Thomas, where he has taught Calculus I and Multi-Variable Calculus. He received his B.A., cum laude, from St. Olaf College and his Ph.D. from the University of Iowa. His specialty is topology and computational mathematics (physical knot theory).

W. Stephen Wilson, Ph.D., is a professor of mathematics at Johns Hopkins University. He received his S.B., S.M., and Ph.D. from M.I.T. His fields are algebraic topology; homotopy theory; complex cobordism; Brown-Peterson homology; and Morava K-theory.

[i] Communicating, reasoning, problem solving, and making connections with mathematical content.

[ii] Numerous studies of the results of the Third International Math and Science Study (TIMSS) found that American teachers are tackling an ever-wider range of math topics each year compared to teachers in countries with higher math achievement. "In other countries, they might spend a month on a topic while we spend days on a topic," says William Schmidt, the U.S. research coordinator for TIMSS. The "inch-deep" coverage makes it harder for students to remember what they learned. "Then next year, since they've forgotten it all, we have to review it." As a result, extensive time is spent each year on the same basic skills.

[iii] U.S. fourth graders use calculators and computers in mathematics class more frequently than do students in most other TIMSS countries. Use of calculators in U.S. fourth-grade mathematics classes is about twice the international average. In the U.S., teachers of 39 percent of the students report having students use calculators in their mathematics classes at least once or twice a week compared with the international average of 18 percent. Internationally, the teachers of two-thirds of the TIMSS students report that they never or hardly ever had students use calculators in their mathematics classes compared with the teachers of one-third of U.S. students. In six of the seven nations that outscore the U.S. in mathematics, teachers of 85 percent or more of the students report that students never use calculators in class. Retrieved on June 25 from http://nces.ed.gov/pubs97/report/97255-2a.asp#i

[iv] Using primary 6 (EM1/EM2) — the more rigorous level when compared to (EM3). Two tracks (same content, slower pace).

[v] "O" is the level required for college admission.