The Mathematics Semester

The Mathematical Education of Elementary School Teachers

A Mathematics – Mathematics Education Partnership at the University of Nebraska-Lincoln

Jim Lewis (and Ruth Heaton)

University of Nebraska-Lincoln

or

What Important Mathematics do Elementary School Teachers "Need to Know" (and) How Should They "Come to Know" Mathematics?

What's the big deal?

Can't any good K-6 school teacher teach children school mathematics?

What is so difficult about the preparation of mathematics teachers?

- Our universities do not adequately prepare mathematics teachers for their mathematical needs in the school classroom. Most teachers cannot bridge the gap between what we teach them in the undergraduate curriculum and what they teach in schools.
- We have not done nearly enough to help teachers understand the essential characteristics of mathematics: its precision, the ubiquity of logical reasoning, and its coherence as a discipline.
- The goal is not to help future teachers learn mathematics but to make them better teachers.

H. Wu

What is so difficult ….?

- For most future elementary school teachers the level of need is so basic, that what a mathematician might envision as an appropriate course is likely to be hopelessly over the heads of most of the students.

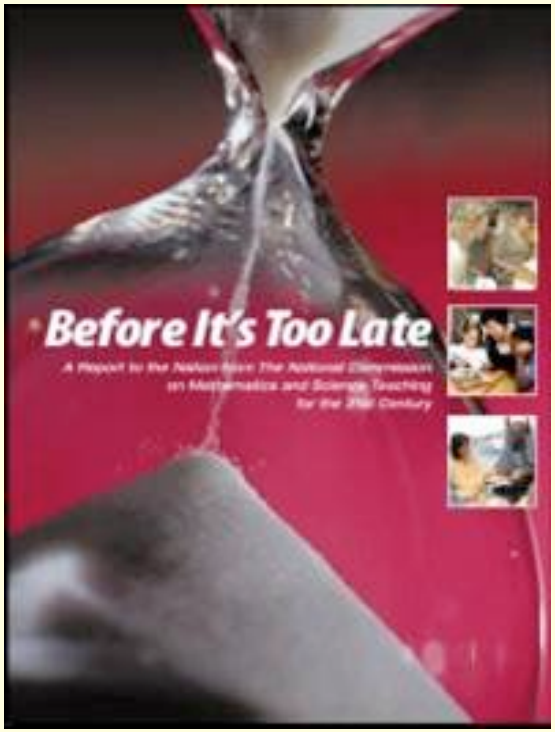
- The mathematics taught should be **connected as directly as possible to the classroom.** This is more important, the more abstract and powerful the principles are. Teachers cannot be expected to make the links on their own.

- Get teacher candidates to believe, that mathematics is something you think about - that validity comes from inner conviction that things make sense, that mathematical situations can be reasoned about on the basis of a few basic principles.

- The goal is to have them develop some flexibility in their thinking, to be able to reason about elementary mathematics.

Roger Howe

Before It's Too Late

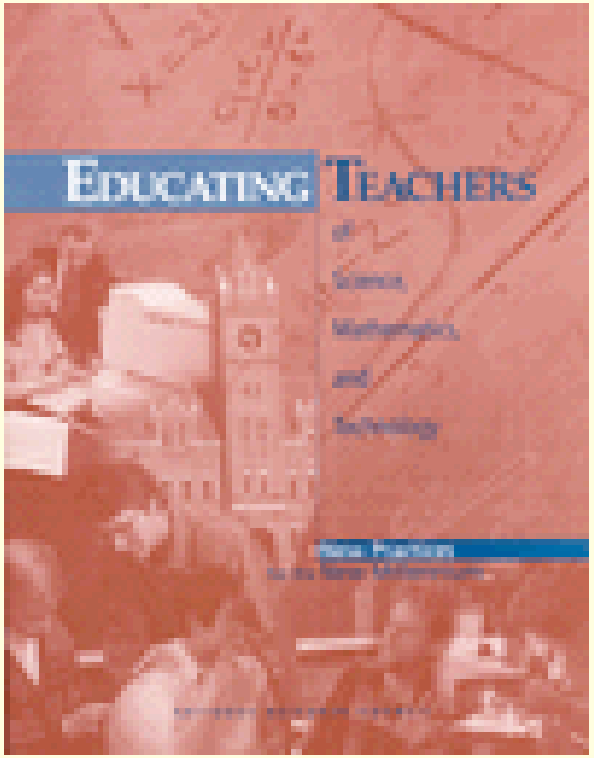


In an age now driven by the relentless necessity of scientific and technological advance, the current preparation that students in the United States receive in mathematics and science is, in a word, unacceptable.

Goal 1: Establish an ongoing system to improve the quality of mathematics and science teaching in grades K-12.

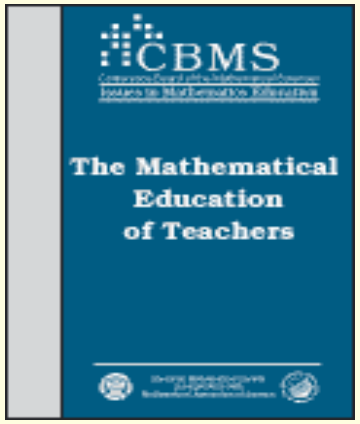
Goal 2: Increase significantly the number of mathematics and science teachers and improve the quality of their preparation.

Educating Teachers of Science, Mathematics, and Technology New Practices for the New Millennium



This National Research Council report recommends:

"a new partnership between K-12 schools and the higher education community designed to ensure high-quality teacher education and professional development for teachers."



The Mathematical Education of Teachers

Recommendations

- Teachers need mathematics courses that develop a deep understanding of the math they teach.
- Elementary teachers should take at least 9 hours of mathematics. Middle grades teachers (5-8) should take at least 21 hours of mathematics.
- Mathematics courses should
  - focus on a thorough development of basic mathematical ideas.
  - develop careful reasoning and mathematical ‘common sense’.
  - develop the habits of mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching.
- The mathematics education of teachers should be based on
  - partnerships between mathematicians, mathematics education faculty and school mathematics teachers.

Math Matters

A NSF-funded CCLI Grant

Project began 01/01/2000

Started with two yearlong cohorts

And then tried two one-semester programs

Implemented The Mathematics Semester

Starting Fall 2003

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The Math Matters Vision

- Create a mathematician – mathematics educator partnership with the goal of improving the mathematics education of future elementary school teachers
- Link field experiences, pedagogy and mathematics instruction
- Create math classes that are both accessible and useful for future elementary school teachers

Barriers to a Successful Partnership

- Math expectations seem to overwhelm students in Elementary Education
- Student evaluations critical of math faculty

Type of Course	Faculty GPA	#Students
– Honors class	3.20	1,367
– All faculty courses	**3.04**	16,693
– Large Lectures	2.88	6,060
– Education Majors	2.48	726

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Comments from a math class for elementary school teachers:

(the course GPA was 2.93)

- This wasn't a course where we learn to teach math. Why do we have to explain our answers.
- I did not like getting a 0 on problems that I attempted. I could have just of left the problem blank then.
- Tests are invalid. They ask questions we have never seen before. It would help if we knew more about the questions on the exams - If examples in class were used on the exams.
- Her way of assessing her class aren't fair.
- Test materials were not consistent or reliable with the material covered in class. Grading was very biased.

Comments from a Contemporary Math class

- (She) does a good job making the subject matter interesting. She always seems very enthusiastic about the class and and actual work. More teachers should be like her.
- (She) is a great teacher with a love for her subject that becomes addictive. It has really been my lucky pick to have gotten her as an instructor.
- (She) made the class exciting. It is obvious she enjoys math and teaching. She was always clear in her expectations and directions.
- This was a very useful class. I also think that (she) is a great teacher.

Barriers to a Successful Partnership

- Students took math courses before admission to Elementary Education Program
- Math for Elementary Education was often taught by graduate students or part-time lecturers
- Cultural differences in how instruction delivered and students assessed
  - Fall 2000 Undergraduate GPA by Dept.
    - Math 2.53 (UNL's lowest)
    - TLTE 3.64 (among highest)

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Mental Math Quiz

(Work all problems in your head.)

1) 48 + 39 =

2) 113 – 98 =

3) 14 x 5 x 7 =

4) is closest to what integer?

5) 4 x 249 =

6) 6(37 + 63) + 18 =

7) .25 x 9 =

8) 12.03 + .4 + 2.36 =

9) 1/2 + 1/3 =

10) 90% of 160 =

11) The sum of the first ten odd positive integers (1+3+5+…+17+19) is equal to what integer?

12) If you buy items (tax included) at $1.99, $2.99 and $3.98, the change from a $10 bill would be?

13) To the nearest dollar, the sale price of a dress listed at $49.35 and sold at 25% off is _____?

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Mental Math Quiz

(Work all problems in your head.)

14) The area of a square of perimeter 20 is ___?

15) The ratio of the area of a circle of radius one to that of a circumscribed square region is closest to?

a) .5, b) .6, c) .7, d) .8, e) .9

16) The average (arithmetic mean) of 89, 94, 85, 90, and 97 is _____ ?

17) If 4/6 = 16/x, then x = _____ ?

18) If 2x + 3 = 25, then x = _____ ?

19) The square root of 75 is closest to what integer?

20) To the nearest dollar, a 15% tip on a restaurant bill of $79.87 is _____ ?

How do our students do on the Mental Math Quiz?

Group	Number	Median Correct	Average Correct
Total	**366**	**12**	**11.7**
– 16 classes			
LPS Wks.	**22**	**13.5**	**13.7**
– Half Middle Level Teachers			

The Mathematics Semester

(For all Elementary Education majors starting Fall 2003)

MATH

- Math 300 – Number and Number Sense (3 cr)

PEDAGOGY

- TEAC 308 – Math Methods (3 cr)
- TEAC 351 – The Learner Centered Classroom (2 cr)

FIELD EXPERIENCE

- TEAC 297b – Professional Practicum Exper. (2 cr)

  (at Roper Elementary School)
  - Students are in Roper Elementary School on Mondays and Wednesdays (four hours/day)
  - Math 300 & TEAC 308 are taught as a 3-hour block on Tuesday and Thursday
  - TEAC 351 meets at Roper on Mondays

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A look inside Math Matters
(and The Mathematics Semester)

- Homework to develop mathematical habits of mind
- Professional/Reflective Writings
- Curriculum Materials
- Number and Number Sense Items
- Working together as a team
- The Curriculum Project
- Activities at Roper
  - Teaching a Math Lesson
  - Child Study
- Learning and Teaching Project

A problem to get started
Making change

What is the fewest number of coins that it will take to make 43 cents if you have available pennies, nickels, dimes, and quarters? After you have solved this problem, provide an explanation that proves that your answer is correct?

How does the answer (and the justification) change if you only have pennies, dimes, and quarters available?

A Typical Weekly Homework All Shook Up

Five couples met one evening at a local restaurant for dinner. Alicia and her husband Samuel arrived first. As the others came in some shook hands and some did not. No one shook hands with his or her own spouse. At the end Alicia noted that each of the other 9 people had shaken the hands of a different number of people. That is, one shook no one's hand, one shook one, one shook two, etc., all the way to one who shook hands with 8 of the people. How many people did Samuel shake hands with?

Professional/Reflective Writings

Purpose: To make connections within and across mathematical, pedagogical, and field experiences through writing.

Professional/Reflective Writings

Dear Math Professors,

We are 1st and 2nd graders in Wheeler Central Public School in Erickson, Nebraska. We love to work with big numbers and have been doing it all year! Every time we read something with a big number in it we try to write it. Then our teacher explains how to write it. We are getting pretty good at writing millions and billions!

We have a problem that we need your help with. We were reading amazing 'Super Mom' facts in a Kid City magazine. It told how many eggs some animals could lay. We came across a number that we don't know. It had a 2 and then a 1 followed by 105 zeros!! We wrote the number out and it stretches clear across our classroom! We know about a googol. We looked it up in the dictionary. A googol has 100 zeros. Then what do you call a number if it has more than 100 zeros? Is there a name for it?

Another problem is that we learned about using commas in large numbers. In the magazine article they used no commas when writing this large number. That confused us. Also, if you write a ‘googol’ with 100 zeros, how do you put the commas in? It doesn’t divide evenly into groups of 3 zeros. There will be one left over.

We appreciate any help you can give us solving this “big” problem. Thank you for your time.

Sincerely,

Mrs. Thompson’s 1st & 2nd graders

Megan Kansier, Mark Rogers

Marcus Witt, Ashley Johnson

clipping from Kid City magazine

Apple Of My Eye

The tiny female apple aphid is a champ as an egg-layer. This insect can lay as many as 21000000000000
00000000000000000000000
00000000000000000000000
00000000000000000000000
00000000000000000000000
0000 eggs in 10 months.

Professional/Reflective Writings

Read “What do Math Teachers Need to Be?” The author is Herb Clemens, a mathematics professor at The Ohio State University, and the article was published in 1991 in *Teaching academic subjects to diverse learners* (pp. 84-96). In this article, Herb Clemens lists what he thinks teachers of mathematics need to be. After reading his article and his meaning and use of these words, where does your own practice of teaching mathematics stand in relationship to what Clemens says mathematics teachers need to be: *unafraid, reverent, humble, opportunistic, versatile, and in control of their math.* On p. 92, Clemens lists four fundamental questions about mathematics teaching that matter to him. If he came to your practicum classroom and watched you teach a math lesson tomorrow, how would he answer his own last question about your practice: Can this teacher teach it [math] with conviction, and with some feeling for its essence? Explain.

Professional/Reflective Writings

Read "Teaching While Leading a Whole-Class Discussion," Chapter 7 from Lampert's book. In this chapter Lampert examines problems of practice that arise while addressing a whole group of students or choosing students to answer questions. As you read the chapter find places in the chapter where you can relate Lampert's writing to your own experiences in the practicum setting while teaching math and maybe even other subjects. Use quotes from the text that connect to your experiences. Explain how and why they relate.

Curriculum Materials

- Sowder, J. et al. (2007). *Reasoning about Numbers and Quantities*. W. H. Freeman. (prepublication copy)
- Schifter, D., Bastable, V., & Russell, S.J. (1999). *Number and operations, part I: Building a system of tens*. Parsippany, NJ: Dale Seymour.
- Reys, Lindquist, Lambdin, Smith, & Suydam (2007). *Helping Children Learn Mathematics*. John Wiley & Sons.
- Lampert, M. (2001). *Teaching problems and the problems of teachin*g. New Haven, CT: Yale University.

Sample Test Items

1) Give an example of one number that you are sure is an irrational number. Explain why you know that it is irrational.
2) Let B = 11232. Factor B into a product of powers of prime numbers. Then factor B^2 into a product of powers of prime numbers.
3) What is the smallest positive integer with exactly 10 factors?
4) Is 2^{50} a factor of 100^{30}? Explain.

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8.881784197 E44

Is 2^{50} a factor of 100^{30}? Explain.

8.881784197 E44

$$\frac{100^{30}}{2^{50}} = \frac{4^{30} * 25^{30}}{2^{50}} = \frac{2^{60} * 5^{60}}{2^{50}} = 2^{10} * 5^{60}$$

"Why is this stuff so hard?"

I believe this test, this class, this subject, are all difficult because they involve thinking in different ways than what we are used to. We have all been conditioned, in our own education; to believe that things are the way they are, and that's all there is to it. We haven't challenged ideas and proofs nearly as much as we should have. Asking "Why" to an idea or trying to understand the reasoning behind something is just not something most of us are used to doing. That's why this stuff is hard.

Miss A

"Why is this stuff so hard?"

I don't have a difficult time with abstract ideas. I love it when we work with new concepts. … I just want you to know that I have almost always been able to figure math problems out and I get VERY frustrated when I get stumped. I am very stubborn like that. Please don't take my temper personally.

Miss J

"Why is this stuff so hard?"

The major problem that I had was my reasoning for the factoring problem. I started off thinking that I should try dividing 2^50 into 100^30, but the large numbers were daunting, so I panicked and tried using my calculator. The answer it gave me did not look pretty, which I think is what triggered my fall down a road of insanity (see my test for more details). Bad, bad calculators ....once you started to explain the problem on the board, I wanted to smack myself in the head for being so silly.

Miss P

Curriculum Project

The goal is to investigate a new mathematical area of the elementary curriculum and consider what teachers need to know as well as what children need to learn the topic in the deep and meaningful ways suggested by the NCTM Standards (2000).

1) Pick a topic: data analysis and probability, geometry, reasoning and proof, or algebra.
2) Read, analyze, and synthesize the mathematical topic in the NCTM Standards.
3) Analyze and synthesize the topic in a set of reform curriculum materials (Everyday Math, Trail Blazers, Investigations, local curriculum).
4) What do teachers need to know to teach this?
5) Create 5 math problems that would help teachers learn. Create 5 math problems that would help children learn.

Math Lesson

You need to teach a math lesson. It should connect to the curriculum in the classroom in which you are working. You should make use of the textbook and other resources from your cooperating teacher and what you are learning and reading in your courses with Jim and Ruth this semester.

Child Study

This assignment will give you some experience watching, listening to, probing, and assessing one child's understanding of several math problems focused on a particular area of mathematics. You will write a report of your interview and suggest instruction for the child based on the information you gathered in the interview session.

Learning and Teaching Project
Area and Perimeter

The task began with a homework problem. It is taken from Reconceptualizing Mathematics: Courseware for Elementary and Middle School Teachers, *Center for Research in Mathematics and Science Education*, 1998.

Is there a relationship between the area and the perimeter of a polygonal shape made with congruent square regions? (For fixed area, find the minimum and maximum perimeter. For fixed perimeter, find the minimum and maximum area.) *Squares must be joined complete-side to complete-side. The outside "boundary" should be a polygon. In particular, this would not permit a shape with a "hole" in the middle.*

A couple of weeks later we told our students:

We want to revisit the “Area and Perimeter” problem. This is to be the basis for a mathematics lesson that you will videotape yourself teaching to one elementary school student.

How can you present this task to the student you will teach? How can you set the stage for the student to understand the problem? How far can the student go in exploring this problem? Remember that you want your student to discover as much as possible for himself (or herself). But there may be some critical points where you need to guide the student over an intellectual “bump” so that he (she) can move on to the next part of the problem.

Finally, produce a report analyzing the mathematics and your teaching experience.

Is the Math Semester a Success?

- The response from our students is very positive.
- Several students are engaged in research projects.
  - Over the past 5 years, 16 students have been involved in an undergraduate research project.
- Our project evaluator reported:
  - significant cohesion among the students; a community of professionals with high standards
  - Cooperating teachers working with the elementary teacher education program assess current or former MM students as better prepared to be a teacher than a control group at a comparable point in their teacher education program.
- When our graduates apply for jobs, many seek recommendations from both of us and we frequently hear that preparation to be an outstanding math teacher was key to their successful job search.

What are we learning?

- Integrate content and pedagogy course offerings.
- Keep expectations of students high.
- Emphasize learning how to learn and offer continued opportunities.
- Build on existing relationships.
- Commitment to the partnership need to be long term.
- Partnership relationships need to extend beyond the relationship of two individuals.

A visit to Carla's 1st Grade Class

- "Carla" was in our first cohort of "Math Matters" students. She volunteered for the program because she considered herself weak in mathematics and she was highly motivated to be a good teacher.
- Two years ago, Dr. Heaton and I visited Carla's 1st Grade Class. The school was located in a "low-income" area of Lincoln.

Carla was teaching a geometry lesson

- The lesson was masterfully crafted. Time was used efficiently. For over an hour the students were eagerly engaged in learning mathematics. Carla's explanations were clear and her use of terminology was careful. Most importantly, the classroom was filled with questions:
  - Why?
  - How do you know?
  - What does that mean?

    etc.

- There was lots of encouragement and plenty of praise for a job well done. One comment stood out. Repeatedly, these 1st grade students were rewarded with the encouragement:

You are so smart!

That's why you are mathematicians!